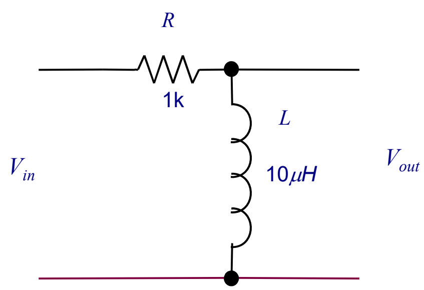
Fourier Transforms

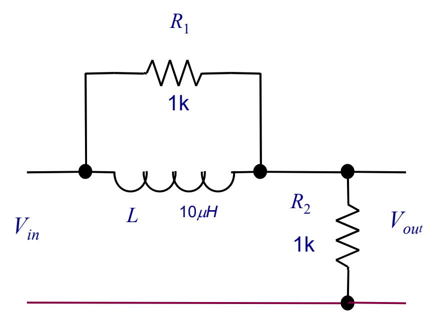
Lecture 9

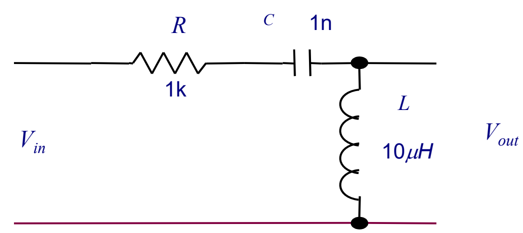
4CT.1-4

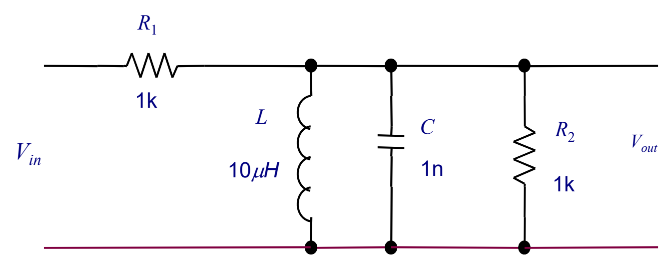
Homework

- Calculate the Fourier Transform of the network function for the following networks:





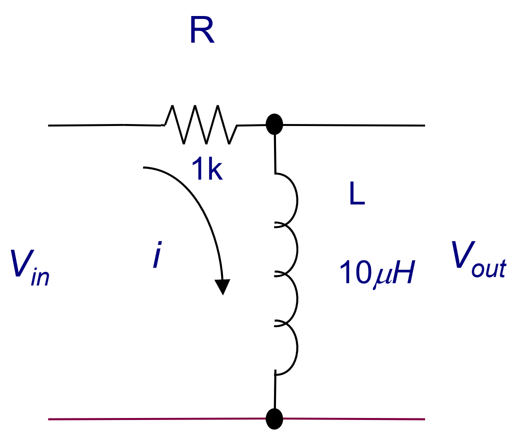




- 4CT.2.2
- 4CT.2.3

Homework Answers #2

- Calculate the Fourier Transform of the network function for the following networks:



$$\Im\{v_{in}(t)\} = V_{in}(j\omega) = \Im\{i(t)R + L\frac{di(t)}{dt}\}$$

$$= R\Im\{i(t)\} + L\Im\{\frac{di(t)}{dt}\} = RI(j\omega) + Lj\omega I(j\omega)$$

$$V_{in}(j\omega) = (R + j\omega L)I(j\omega)$$

$$\Im\{v_{out}(t)\} = V_{out}(j\omega) = \Im\{L\frac{di(t)}{dt}\} = j\omega LI(j\omega)$$

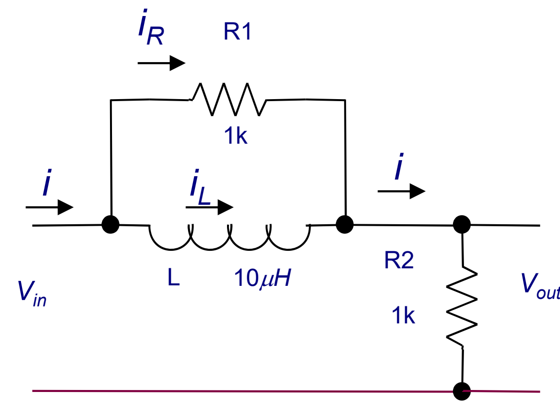
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega 10\times 10^{-6}}{10^3 + j\omega 10\times 10^{-6}}$$

$$= \frac{j\omega 10^{-5}}{10^3 + j\omega 10^{-5}} = \frac{j\omega 10^{-8}}{1 + j\omega 10^{-8}}$$

$$= \frac{\omega / 10^8}{\sqrt{1 + (\omega/10^8)^2}} \angle \frac{\pi}{2} - \tan^{-1}(\omega/10^8)$$

Homework Answers #2

- Calculate the Fourier Transform of the network function for the following networks:



$$v_{out}(t) = i(t)R_2$$

$$V_{out}(j\omega) = I(j\omega)R_2$$

$$i(t) = i_R(t) + i_L(t) = \frac{v_{in}(t) - v_{out}(t)}{R_1} + \frac{1}{L}\int [v_{in}(t) - v_{out}(t)]dt$$

$$\Im\{i(t)\} = I(j\omega) = \Im\{\frac{v_{in}(t) - v_{out}(t)}{R_1}\} + \Im\{\frac{1}{L}\int [v_{in}(t) - v_{out}(t)]dt\}$$

$$I(j\omega) = \frac{V_{in}(j\omega) - V_{out}(j\omega)}{R_1} + \frac{V_{in}(j\omega) - V_{out}(j\omega)}{j\omega L}$$

$$\frac{V_{out}(j\omega)}{R_2} = \frac{V_{in}(j\omega) - V_{out}(j\omega)}{R_1} + \frac{V_{in}(j\omega) - V_{out}(j\omega)}{j\omega L}$$

$$\{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{j\omega L}\}V_{out}(j\omega) = \{\frac{1}{R_1} + \frac{1}{j\omega L}\}V_{in}(j\omega)$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{\frac{1}{R_1} + \frac{1}{j\omega L}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{j\omega L}} = \frac{\frac{j\omega L + R_1}{j\omega L(R_1)}}{\frac{j\omega LR_1 + j\omega LR_2 + R_2R_1}{j\omega L(R_1)(R_2)}}$$

$$= \frac{(j\omega L + R_1)R_2}{j\omega LR_1 + j\omega LR_2 + R_2R_1}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R_2}{R_2 + \frac{R_1 j\omega L}{R_1 + j\omega L}} = \frac{10^3}{10^3 + \frac{(j\omega 10\times 10^{-6})10^3}{10^3 + (j\omega 10\times 10^{-6})}}$$

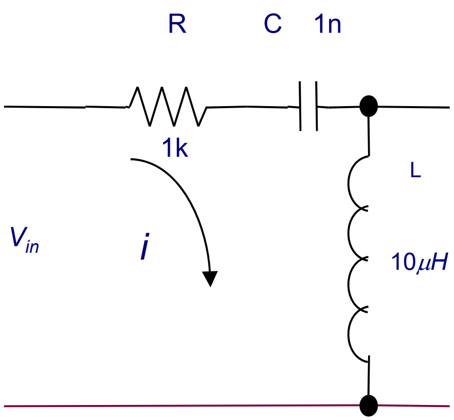
$$= \frac{1}{1 + \frac{(j\omega 10\times 10^{-6})}{10^3 + (j\omega 10\times 10^{-6})}} = \frac{10^3 + (j\omega 10\times 10^{-6})}{10^3 + (j\omega 10\times 10^{-6}) + (j\omega 10\times 10^{-6})}$$

$$= \frac{10^3 + j\omega 10\times 10^{-6}}{10^3 + 2j\omega 10\times 10^{-6}} = \frac{1 + j\omega 10^{-2}}{1 + j2\omega 10^{-2}}$$

$$= \frac{\sqrt{1 + \omega^2 10^{-4}}}{1 + 4\omega^2 10^{-4}} \angle \tan^{-1}(\omega 10^{-2}) - \tan^{-1}(2\omega 10^{-2})$$

Homework Answers #3

- Calculate the Fourier Transform of the network function for the following networks:



$$\Im\{v_{in}(t)\} = V_{in}(j\omega) = \Im\{Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt\}$$

$$= RI(j\omega) + j\omega LI(j\omega) + \frac{1}{j\omega C}I(j\omega)$$

$$= [R + j\omega L + \frac{1}{j\omega C}]I(j\omega)$$

$$\Im\{v_{out}(t)\} = V_{out}(j\omega) = \Im\{L\frac{di(t)}{dt}\} = j\omega LI(j\omega)$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{-\omega^2 LC}{(1-\omega^2 LC) + j\omega CR}$$

$$= \frac{-\omega^2(10\times 10^{-6})(10^{-9})}{[1-\omega^2(10\times 10^{-6})(10^{-9})] + j\omega 10^{-9}(10^3)}$$

$$= \frac{-\omega^2 10^{-14}}{[1-\omega^2 10^{-14}] + j\omega 10^{-6}} = \frac{-\omega^2 10^{-14}}{\sqrt{[1-\omega^2 10^{-14}]^2 + \omega^2 10^{-12}}} \angle -\tan^{-1}(\frac{\omega 10^{-6}}{[1-\omega^2 10^{-14}]})$$

Homework Answers #4

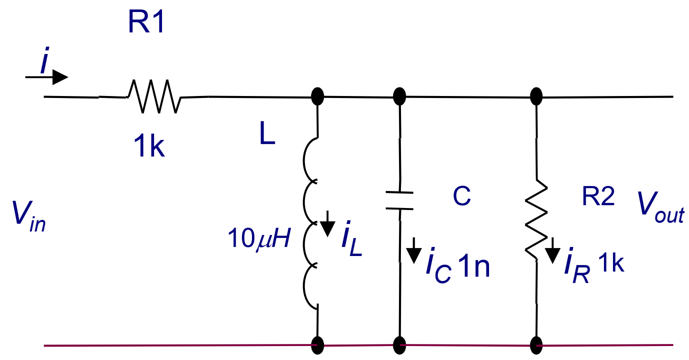
- Calculate the Fourier Transform of the network function for the following networks:

$$\Im\{i(t)\} = I(j\omega) = \Im\{i_L(t) + i_C(t) + i_R(t)\} = \Im\{\frac{1}{L}\int v_{out}(t)dt + C\frac{dv_{out}(t)}{dt} + \frac{v_{out}(t)}{R_2}\}$$

$$I(j\omega) = \frac{1}{j\omega L}V_{out}(j\omega) + j\omega C V_{out}(j\omega) + \frac{V_{out}(j\omega)}{R_2} = [\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2}]V_{out}(j\omega)$$

$$\Im\{v_{in}(t)\} = V_{in}(j\omega) = \Im\{i(t)R_1 + v_{out}(t)\} = I(j\omega)R_1 + V_{out}(j\omega)$$

$$V_{in}(j\omega) = [(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2})R_1 + 1]V_{out}(j\omega)$$

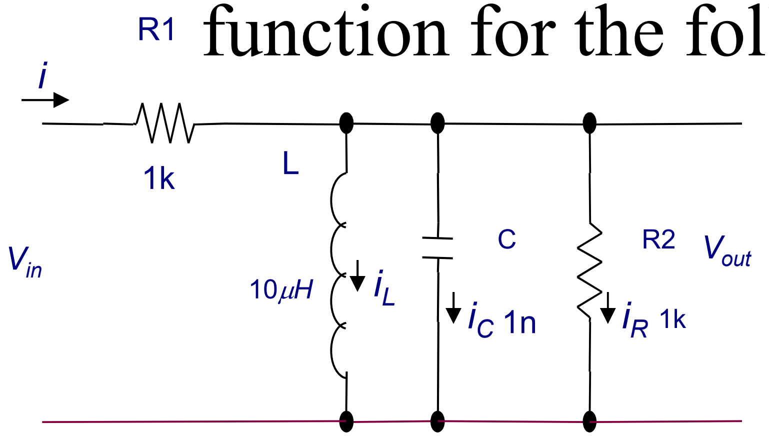


$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2})R_1 + 1} = \frac{\frac{1}{(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2})}}{R_1 + \frac{1}{(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2})}}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L \,\|\, R_2 \,\|\, \frac{1}{j\omega C}}{R_1 + j\omega L \,\|\, R_2 \,\|\, \frac{1}{j\omega C}}$$

Homework Answers #4

- Calculate the Fourier Transform of the network function for the following networks:



$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L \,\|\, R_2 \,\|\, \frac{1}{j\omega C}}{R_1 + j\omega L \,\|\, R_2 \,\|\, \frac{1}{j\omega C}}$$

$$j\omega L \,\|\, R_2 \,\|\, \frac{1}{j\omega C} = \frac{1}{1/j\omega L + 1/R_2 + j\omega C}$$

$$= \frac{1}{\frac{R_2 + j\omega L}{j\omega R_2 L} + j\omega C} = \frac{j\omega R_2 L}{R_2(1-\omega^2 LC) + j\omega L}$$

$$= \frac{j\omega 10^3 (10\times 10^{-6})}{10^3(1-\omega^2(10\times 10^{-6})(10^{-9})) + j\omega(10\times 10^{-6})}$$

$$= \frac{j\omega 10^{-5}}{[1-\omega^2 10^{-14}] + j\omega 10^{-8}}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{\frac{j\omega 10^{-5}}{[1-\omega^2 10^{-14}] + j\omega 10^{-8}}}{10^3 + \frac{j\omega 10^{-5}}{[1-\omega^2 10^{-14}] + j\omega 10^{-8}}}$$

$$= \frac{j\omega 10^{-8}}{[1-\omega^2 10^{-14}] + j2\omega 10^{-8}}$$

$$= \frac{\omega 10^{-8}}{\sqrt{[1-\omega^2 10^{-14}]^2 + 4\omega^2 10^{-16}}} \angle 90^\circ - \tan^{-1}\left(\frac{2\omega 10^{-8}}{[1-\omega^2 10^{-14}]}\right)$$

4CT.2.2

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

$$X(F) = X\delta(F - F_o) = \frac{1}{2} A e^{j\phi} \delta(F - F_o)$$

$$x(t) = \int_{-\infty}^{\infty} \frac{1}{2} A e^{j\phi} \delta(F - F_o) e^{j2\pi Ft} dF = \frac{1}{2} A e^{j\phi} \int_{-\infty}^{\infty} \delta(F - F_o) e^{j2\pi Ft} dF$$

$$= \frac{1}{2} A e^{j\phi} e^{j2\pi F_o t} = \frac{1}{2} A e^{j(2\pi F_o t + \phi)}$$

4CT.2.3

$$x(t) = \int_{-\infty}^{\infty} X(F)e^{j2\pi Ft}dF$$

$$X(F) = X\delta(F - F_o) + X^*\delta(F + F_o) = \frac{1}{2}Ae^{j\phi}\delta(F - F_o) + \frac{1}{2}Ae^{-j\phi}\delta(F + F_o)$$

$$x(t) = \int_{-\infty}^{\infty} [\frac{1}{2}Ae^{j\phi}\delta(F - F_o) + \frac{1}{2}Ae^{-j\phi}\delta(F + F_o)]dF$$

$$= \frac{1}{2}Ae^{j\phi}\int_{-\infty}^{\infty} \delta(F - F_o)e^{j2\pi Ft}dF + \frac{1}{2}Ae^{-j\phi}\int_{-\infty}^{\infty} \delta(F + F_o)e^{j2\pi Ft}dF$$

$$= \frac{1}{2}Ae^{j\phi}e^{j2\pi F_o t} + \frac{1}{2}Ae^{-j\phi}e^{-j2\pi F_o t}$$

$$= \frac{1}{2}Ae^{j(2\pi F_o t + \phi)} + \frac{1}{2}Ae^{-j(2\pi F_o t + \phi)} = A\cos(2\pi F_o t + \phi)$$